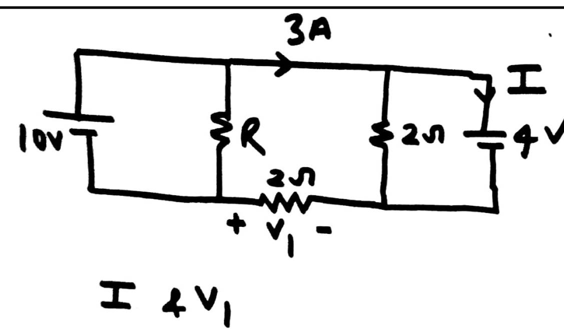
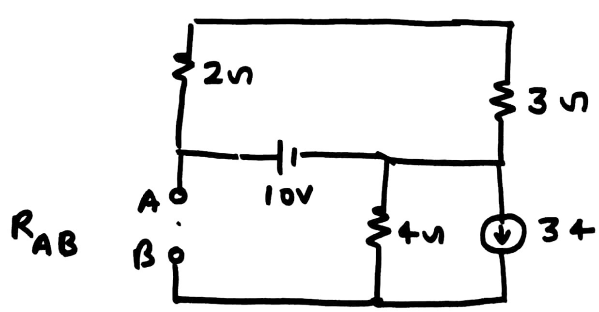
Linear Circuit Theory

L05: Concepts of Mutual inductance

- Solution of last lecture question
- Concepts of mutual inductance series parallel connection
- Questions based on mesh analysis

Tutorial problem





Calculate all node voltage and loop current

Unknown, $I_1\ I_2\ I_3$
$V_A,\ V_B,\ V_C,\ V_1$

$V_A - V_C = 10$

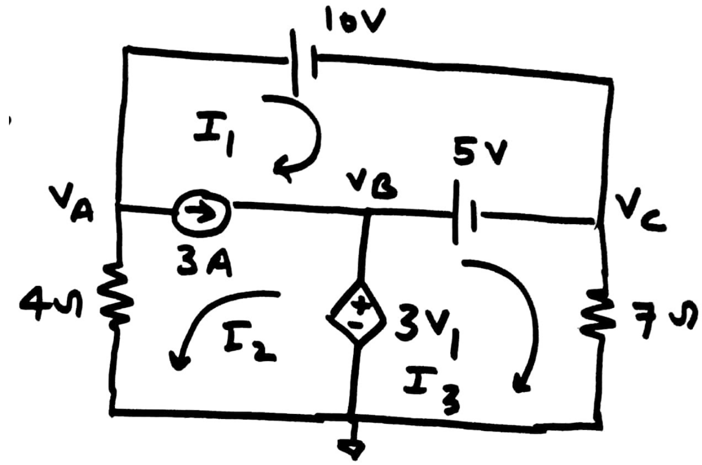
$V_B - V_C = 5$

$V_A = -V_1 = 2I_2$

$V_B = -3V_1$

$I_1 + I_2 = -3$

$10 + 7I_3 + V_1 = 0$

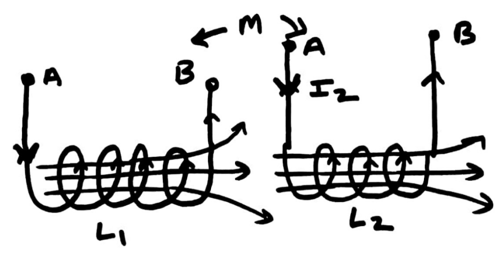


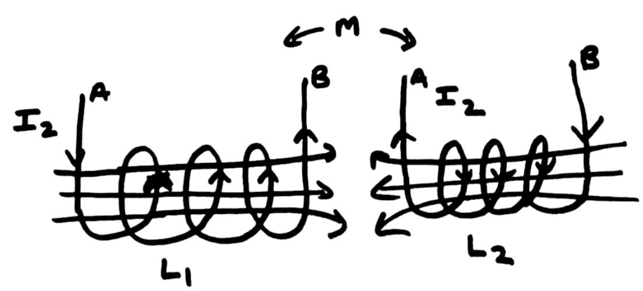
Concept of mutual inductance

$$V_1 = L_1 \frac{\partial I_1}{\partial t} = L_1 s I_1$$

$$V_2 = L_2 \frac{\partial I_2}{\partial t} = L_2 s I_1$$

$$M = k\sqrt{L_1 L_2}$$



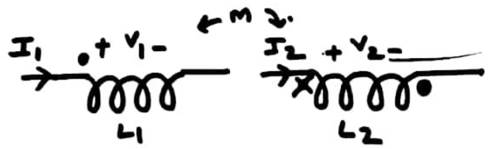


Voltage current relation in case of coupling

$$V_1 = L_1 \frac{\partial I_1}{\partial t} + M \frac{\partial I_2}{\partial t}$$

$$V_2 = L_2 \frac{\partial I_2}{\partial t} + M \frac{\partial I_1}{\partial t}$$

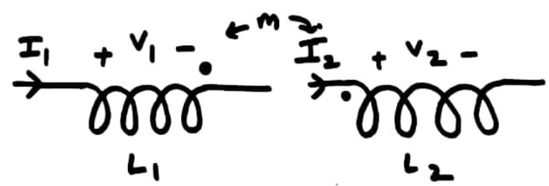
$$V_1 = L_1 s I_1 + M s I_2$$
$$V_2 = L_2 s I_2 + M s I_1$$



$$V_1 = L_1 \frac{\partial I_1}{\partial t} - M \frac{\partial I_2}{\partial t}$$

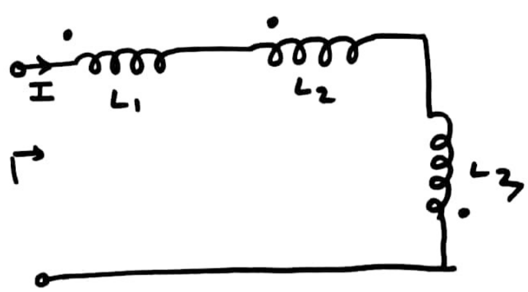
$$V_2 = + L_2 \frac{\partial I_2}{\partial t} - M \frac{\partial I_1}{\partial t}$$

$$V_1 = L_1 s I_1 - M s I_2, \quad V_2 = L_2 s I_2 - M s I_1$$



Equivalent inductance (Series connection)

M_{12}, M_{23}, M_{31}



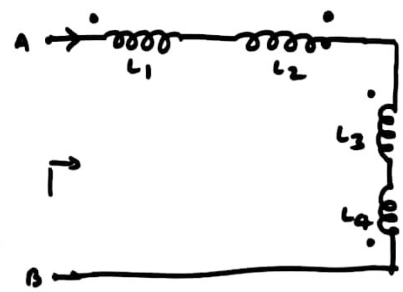
$$V = L_1 \frac{\partial I}{\partial t} + L_2 \frac{\partial I}{\partial t} + L_3 \frac{\partial I}{\partial t} + \pm 2M_{12} \frac{\partial I}{\partial t} - 2M_{23} \frac{\partial I}{\partial t} - 2M_{31} \frac{\partial I}{\partial t}$$

$$L = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} - 2M_{31}$$

Problem on Equivalent inductance

$L_1 = L_2 = L_3 = L_4 = L = 1\ mH$

$M_{12} = M_{23} = M_{31} = M_{41} = M_{24} = M_{34} = 2H$



$L = L_1 + L_2 + L_3 + L_4 +$
$\quad -2M_{12} + 2M_{13} - 2M_{14}$
$\quad -2M_{23} + 2M_{24} - 2M_{34}$

$L_e = 4L - 4M$

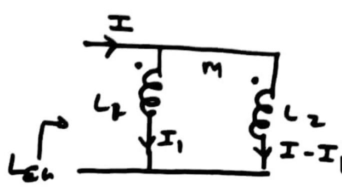
$L_e = 4 - 4 \times 2 = -4\ mH$

Equivalent inductance in parallel case

$V = s L_{eq} I$

$V = s L_1 I_1 + M s (I - I_1) \quad -1$

$V = s L_2 (I - I_1) + M s I_1 \quad -2$



$s L_1 I_1 + M s (I_1 - I_1) = s L_2 (I - I_1) + M s I_1 \quad - 3$

$$I_1 = \frac{I(L_2 - M)}{L_1 + L_2 - 2M} \quad -4$$

Eq - 1 $\quad V = s(L_1 - M) I_1 + M s I$

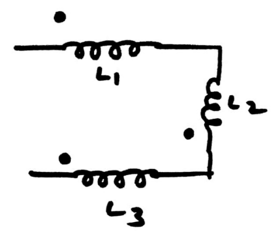
$$V = s \frac{(L_1 - M)(L_2 - M)}{L_1 + L_2 - 2M} I + M s I$$

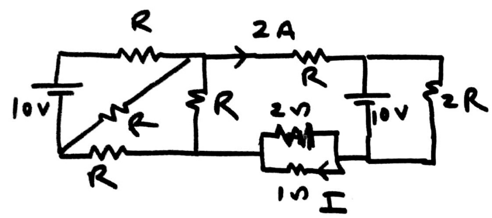
$$L_{eq} = \frac{L_1 L_2 - M L_2 - M L_1 + M^2}{L_1 + L_2 - 2M} + M$$

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}, \qquad L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

Numerical Examples

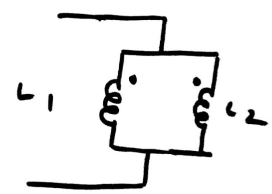
$L_1 = 1H$ $\quad M_{12} = 2H$
$L_2 = 2H$ $\quad M_{23} = 1H$
$L_3 = 3H$ $\quad M_{31} = 1H$





$I = ?$

$L_1 = 1H$
$L_2 = 3H$
$M_{12} = 4H$



Next lecture

- Network Theorems (Thevnins, Norton's Maximum power transfer)
- Numerical on these theorem

Thank you