Head of the Department Mathematics Institute of Applied Sciences & Humanilies GLA University, Mathura

Subjects of Ph.D. Course Work

S. No.	Nature	Subject Code	Subject Name		Cr.
1	Compulsory	PMG 1001	RESEARCH METHODOLOGY	3-0-1-0	4
2	Compulsory	PREC 0010	RESEARCH AND PUBLICATION ETHICS	2-0-0-0	2
Bou	quet of Electi	ves (ANY TW	O as per the research area):		
1		PMAE 0010	COMPUTER SKILLS, TENSOR AND RIEMANNIAN GEOMETRY	4-0-0-0	4
1	Electives	PMAE 0011	RELATIVITY AND COSMOLOGY	4-0-0-0	4
2		PMAE 0020	INVENTORY MODELING	4-0-0-0	4
2	Electives	PMAE 0021	RANDOM VARIABLES AND SOFTWARE TOOLS	4-0-0-0	4
3		PMAE 0030	PARTIAL DIFFERENTIAL EQUATIONS: METHODS AND APPLICATIONS	4-0-0-0	4
3	Electives	PMAE 0031	FRACTIONAL DIFFERENTIAL EQUATIONS	4-0-0-0	4
		PMAE 0040	COMPUTER BASED NUMERICAL TECHNIQUES AND SOFT COMPUTING	4-0-0-0	4
4		PMAE 0041	APPLIED STATISTICS AND OPTIMIZATION TECHNIQUES	4-0-0-0	4
4	Electives	PMAE 0042	STATISTICAL COMPUTATION AND SIMULATION WITH R	4-0-0-0	4
		PMAE 0043	ADVANCED DEEP LEARNING	4-0-0-0	4
		PMAE 0050	DIFFERENTIAL GEOMETRY OF MANIFOLDS	4-0-0-0	4
-		PMAE 0051	RIEMANN-FINSLER GEOMETRY	4-0-0-0	4
5	Electives	PMAE 0052	ALGEBRAIC GRAPH THEORY	4-0-0-0	4
		PMAE 0053	ADVANCED ALGEBRAIC GRAPH THEORY	4-0-0-0	4
(Els et :	PMAE 0060	FLUID DYNAMICS AND SOFTWARE TOOLS	4-0-0-0	4
6	Electives	PMAE 0061	COMPUTATIONAL METHODS FOR FLUID DYNAMICS	4-0-0-0	4
_		PMAE 0070	FOUNDATIONS OF COMPLEX ANALYSIS-I	4-0-0-0	4
7	Electives	PMAE 0071	FOUNDATIONS OF COMPLEX ANALYSIS-II	4-0-0-0	4

PMG1001: Research Methodology

Objective:

To familiarize the scholars with the concept and the techniques of research methodology applicable to develop broad comprehension of research area.

UNIT – I	Introduction to research-motivation and objectives, Uniqueness of Ph.D. Thesis research, Research Philosophy, structure of research (The research hourglass) research process, research design, different types of research design (Exploratory, descriptive, experimental). Analysis of Literature Review – Primary and Secondary Sources, Web sources –critical Literature Review, Hypothesis – Different Types – Significance – Development of Working Hypothesis, Null hypothesis, Research Methods: Scientific method vs Arbitrary Method, Logical Scientific Methods: Deductive, Inductive,	(15 Sessions)	
	I TERM EXAMINATION		
UNIT – II	Methods of Collecting Data : Observation, field investigations, Direct studies– Reports, Records or Experimental observations., Measurement and Scaling, Questionnaire Design, Concept of Reliability and Validity,, Population and sample, fitting the		
	II TERM EXAMINATION		
UNIT – III	Data Analysis- Various Statistical tools and techniques (Measure of Central Tendency & Dispersion, correlation, regression, Parametric Tests & Non-Parametric tests, factor analysis, cluster analysis), Simulation and numerical methods, Concepts of Artificial intelligence and Associated Techniques. Introduction to Intellectual Property rights, Ethics in Research, Plagiarism – definition, forms and consequences.	(15 Sessions)	
	1 Tovision for presentations / assignments / case analysis in additional sessions		

Note:

- a. Relevant Case Studies and research papers should be discussed and presented in the class (Background and interest areas of the students should be considered while selecting the Cases).
- b. Practical problems should be discussed, analyzed and solved wherever applicable.

Students may be exposed to relevant software's like SPSS, MATLAB etc. used for Data analysis keeping in mind the background of the student.

Suggestive Readings:

- Panneer Selvam Research Methodology, Prentice Hall of India, Edition 2008.
- Cooper and Schindler Business Research Methods, Tata McGraw Hill, 9th Edition.
- Zikmund William G. Business Research Methods, 7th Edition, Cengage Learning India Pvt. Ltd., New Delhi
- Saunders M., Lewis P., Thornhills A. Research Methods for Business Students, Pearsons Education, New Delhi 2005.
- Malhotra & Dash Marketing Research, Pearsons Education, New Delhi
- Hair, J. F., Black, W. C., Babin, B. J. and Anderson, R. E.- Multivariate Data Analysis, 7th Edition, Prentice Hall, India 2010.

Learning Outcomes:

- Scholars will be able to understand the concept of research, research philosophy and research structure.
- M Able to understand the importance of literature review and can independently do it.
- Working knowledge of different statistical fundamentals and tools.
- Aware with different style of referencing.

Course Syllabus Research and Publication Ethics: PREC 0010

Credits: 02

Semesters I

L-T-P: 2-0-0

Objectives:

- The course aims to develop a philosophical understanding.
- It develops the logical understanding and ethical decision making
- It expands the rationality and critical thinking to improvise the problem-solving.
- It is to cultivate the scientific temperament, philosophical approach and ethical attitude which are much needed for quality researches.
- It is to give an exposure to the scholars of various fields to understand the philosophy of research, publication ethics, understanding scientific conduct and publication misconduct.
- It is to enable scholars to understand various concepts relating open access publishing, ethical tools, using research database and metrics.

Module	Content	Teaching
No.		Hours
RPE 01	Philosophy and Ethics Introduction to Philosophy: definition, nature and scope, concept, branches	4 hrs
	Ethics: Definition, moral philosophy, nature of moral judgments and reactions.	
RPE02	Scientific Conduct Ethics for science and research, Intellectual honesty and research integrity, Scientific misconducts: Falsification, Fabrication and Plagiarism (FFP), Redundant publications: duplicate and overlapping publications, salami slicing, Selective reporting and misrepresentation of data	4 hrs
RPE03	Publication Ethics	7 hrs
	Publication ethics: definition, introduction and importance	
	Best practices/standards-setting initiatives and guidelines: Committee on Publication Ethics (COPE):, World Association of Medical Editors (WAME) etc.	
	Conflicts of interest, Publication misconduct: Definition, concept, problems that lead to unethical behavior and vice versa, types, Violation of publication ethics, authorship and contributorship, Identification of publication misconduct, complaints and appeals, Predatory publishers and journals	
RPE04	Open Access Publishing (4hrs)	4 hrs
	Open access publications and initiatives, SHERPA/RoMEO online resource to check publisher copyright & self-archiving policies, a	

	Software tool to identify predatory publications developed by SPPU: UGC-CARE list of journals, Journal finder/journal suggestion tools viz. JANE, Elsevier Journal Finder, Springer Journal Suggester, etc.	
REP05	 Publication Misconduct, Group discussions (2 hrs) Subject-specific ethical issues, FFP, authorship, Conflicts of interest, Complaints and appeals: examples and fraud from India and abroad, Software tools (2 hrs) Use of reference management software like Mendeley, Zotero etc. and anti-plagiarism software like Turnitin, Urkund 	
REP06	Databases and research metrics (7 hrs) Databases (4hrs): Indexing databases, Citation databases: Web of Science, Scopus etc. Research Metrics (3hrs): Impact factor of a journal as per Journal Citation Report, SNIP, SJR, IPP, Cite-Score, Metrics: h- index, g-index,i-10 index, altmetrics	7hrs

Outcomes:

- The scholars will be much competent in understanding the ideas of research.
- They will develop a philosophical and ethical attitude to understand the importance of the research.
- They will have the scientific temperament to deal with the research issues.
- They will avoid misconduct in research and publication.
- It will enable them to use various tools, software, databases and research metrics.
- They will be informed, scientific and ethical in their research attitude and will be equipped with the specialized knowledge to use software and tools.

References:

- 1. Think: A Compelling Introduction to Philosophy, by Simon Blackburn.
- 2. <u>https://www.youtube.com/watch?v=IvwkMxgahA4</u>
- 3. Logical Positivism: <u>https://www.youtube.com/watch?v=VAqicPNeKYo</u>
- 4. <u>https://www.youtube.com/watch?v=1A_CAkYt3GY</u>
- 5. <u>https://www.slideshare.net/LorraineSiscar/nature-of-philosophy</u>
- 6. https://plato.stanford.edu/entries/vienna-circle/
- 7. <u>https://plato.stanford.edu/entries/logical-empiricism/</u>
- 8. https://lucidphilosophy.com/1019-2/
- 9. <u>https://www.slideshare.net/clhendricksbc/judith-jarvis-thomson-the-trolley-problem</u>
- 10. Publication Misconduct, <u>https://www.youtube.com/watch?v=fGgwNClHyCo</u>
- 11. Publication Ethics & Scientific Writing https://www.youtube.com/watch?v=QsMylCHYm4E
- 12. Open Access Publishing: https://www.youtube.com/watch?v=n1ZySivYQ3w



Head of the Department Mathematics Institute of Applied Sciences & Humanilies GLA University, Mathura

PMAE 0010: COMPUTER SKILLS, TENSOR AND RIEMANNIAN GEOMETRY

Course Objectives: To make the students understand the concept of tensors and Riemannian geometry and know the software packages by giving more emphasis to their applications.

Credits: 04

Semester I

L-T-P: 4-0-0

Module No.	Contents	Teaching Hours (Approx.)
Ι	 Computer Skills I: Introduction to Latex, MATHEMATICA and MAPLE. Tensor and Riemannian Geometry I: Tensor calculus, n-dimensional space V_n, Superscript and subscript, Transformation of coordinates, Transformation law of tensor, Product of two tensors, Contraction, Trace of a tensor, Quotient law, Metric tensor and Riemann space. Computer Skills II: Introduction to Excel, Chart, Functions. Power point presentation, Introduction to C language. Tensor and Riemannian Geometry II: Associated and Reciprocal or conjugate tensor, Symmetric and anti symmetric tensor, Tensor density, Levi-Civita tensor, Christoffel symbols, Law of transformation of Christoffel symbols, Covariant differentiation, Riemannian Affine connection, Covariant derivative of a vector density, Riemannian metric, Geodesics, Null geodesics, Tensor form of gradient, divergence, Laplacian and curl. Tensor and Riemannian Geometry III: Intrinsic derivative, Riemannian and normal coordinates, Gaussian coordinates, Parallel transport, Geodesics as auto parallel curves, Parallel propagation, Riemann curvature tensor Rⁱ_{ljk}, Covariant curvature tensor, Symmetric properties of Rⁱ_{ljk}, Covariant curvature tensor, Bianchi identities, Conformal curvature tensor, Algebraic classification of the conformal curvature tensor, Conformal invariance, Geodesic deviation, Lie derivative. 	45

Learning Outcomes:

After studying these topics, the students will be able to

- Use software packages in research
- Learn different types of tensors
- > Find Christoffel symbols and use them in the computation of different curvature tensors
- > Understand covariant differentiation, Bianchi identities and their applications

Text Books:

- S. R. Roy and Raj Bali, Theory of Relativity, Jaipur Publishing House, 2008.
- S. K. Srivastava, General Relativity and Cosmology, PHI Pvt. Ltd., 2008.
- > Leslie Lamport, A Document preparation System: Latex, Addison-Wesley Professional, 1994.
- Y. K. Singh and B. B. Chaudhary, MATLAB Programming, PHI, 2007.

Reference Book:

J. V. Narlikar, Cosmology, Cambridge University Press, 2002.



Head of the Department Mathematics Institute of Applied Sciences & Humanities GLA University, Mathura

PMAE 0011: RELATIVITY AND COSMOLOGY

Course Objectives: To make the students understand the concept of general relativity and cosmology by giving more emphasis to their applications.

Credits	: 04 Semester II	L-T-P: 4-0-0
Module	Contents	Teaching Hours
No.		(Approx.)
Ι	Special Theory of Relativity: Inertial and non-inertial frames, Special and General Galilean transformations, Lorentz transformation and its geometrical interpretation, Transformation formula for mass, density, momentum, energy and force, Minkowski-space, Relativistic equation of motion, Four vectors and tensors in Minkowski space, Lagrangian and Hamiltonian formulation of Relativistic Mechanics. General Relativity: Principles of equivalence and general covariance, Mach's Principle, Einstein's field equations, Energy momentum tensors, Gravitational equations, Vectors and tensors, Experimental tests of general relativity, Alternatives theories of gravitations, FRW model, Schwarzschild solution, Cosmological solutions in Brans-Dicke Theory, Kaluza's five dimensional theory, Cosmological models, Singularity in cosmological models. Cosmology: Static cosmological models, Newtonian cosmology, Einstein universe, Expanding universe, Friedmann models, Cosmological models with non-zero cosmological term, The early universe, The inflationary universe, Primordial black holes, Dark energy and dark matter, Observational constraints on cosmological parameters, Standard cosmology.	45

Learning Outcomes:

After studying these topics, the student will be able to

- > Understand the basic principles of cosmology
- > Know the significance the Einstein's theories of special and general relativity
- > Deal with the cosmological models
- Learn various theories of gravitation

Text Books:

- S. R. Roy & Raj Bali, Theory of Relativity, Jaipur Publishing House, 2008.
- > J. V. Narlikar, An Introduction to Cosmology, Cambridge University Press, 2002.

- S. Weinberg, Cosmology, Oxford University Press, 2008.
- S. K. Srivastava, General Relativity and Cosmology, PHI Pvt. Ltd., 2008.



Head of the Department Mathematics Institute of Applied Sciences & Humanities GLA University, Mathura

Course Curriculum (Session 2020-2021) [Ph.D. Course Work]

PMAE 0020: INVENTORY MODELING

Course Objectives: To make the students understand the various inventory models with deterministic & probabilistic demands, price breaks and different demand functions.

Credits: 04

Semester I

L-T-P: 4-0-0

Contents	Teaching Hours
	(Approx.)
Inventory Models with Deterministic Demand:	
(1) Demand rate uniform, production rate infinite,	
(2) Demand rate non uniform and production rate	
5	
shortages allowed.	
Inventory Models with Probabilistic Instantaneous	
Demand:	
	45
	10
Demand:	
(1) Continuous demand, setup cost zero, stock level	
Inventory Models with Price Breaks:	
Purchase inventory models with one and two price	
breaks	
demand.	
	Basics of Inventory Models (IM): Necessity of IM, Inventory costs, Basic definitions, Classification of IM.Inventory Models with Deterministic Demand: (1) Demand rate uniform, production rate infinite, (2) Demand rate non uniform and production rate infinite, (3) Demand rate uniform and production rate infinite, (4) Demand rate uniform, production rate infinite & shortages allowed, (5) Demand rate uniform, production rate finite& shortages allowed. Inventory Models with Probabilistic Instantaneous Demand: (1) Instantaneous demand, setup cost zero, stock level discrete and lead time zero (2) Instantaneous demand, setup cost zero, stock level continuous and lead time zero (1) Continuous demand, setup cost zero, stock level discrete and lead time zero (2) Continuous demand, setup cost zero, stock level discrete and lead time zero (2) Continuous demand, setup cost zero, stock level discrete and lead time zero (3) Continuous demand, setup cost zero, stock level discrete with lead time zero (3) Continuous demand, setup cost zero, stock level discrete with lead time. Inventory Models with Price Breaks: Purchase inventory models with one and two price breaksInventory Models with price and time dependent demand, Inventory models with price, time and stock dependent demand, Inventory models with price, time and stock

Learning Outcomes:

After studying these topics, the students will be able to

- Learn inventory model and its types
- > Understand the inventory models with uniform and non-uniform demand rate
- Know the inventory models with instantaneous/ continuous demand with discrete/ continuous stock level and lead time zero
- > Minimize the total inventory holding costs and ordering costs



Text Books:

- P. K. Gupta & D. S. Hira, Operations Research, S. Chand Publication, 2014.
- Sven Axsäter, Inventory Control, Springer, 2000.

Reference Book:

E. L. Porteus, Foundations of Stochastic Inventory Theory, Stanford University Press, 2002.



PMAE 0021: RANDOM VARIABLES AND SOFTWARE TOOLS

Course Objectives: To make the students understand the concepts of probability distribution, use of software tools and LaTeX.

Credits: 04

Semester I/II

L-T-P-J: 4-0-0-0

Module No.	Contents	Teaching Hours
		(Approx.)
Ι	Random Variables - Discrete and Continuous, Cumulative Distribution and Probability Density Function, Discrete Random Variables, Probability Mass Function and Cumulative Distribution Function, Expectation of Variables, Moments and Variance, Skewness and Kurtosis, Degenerate and Discrete Uniform Distributions, Bernoulli and Binomial Distribution, Poisson Distribution, Geometric Distribution, Continuous Random Variables and Uniform Distribution, Normal Distribution. Software Tools - Introduction to softwares: Mathematica, Matlab, SPSS and R. Plotting of functions, Symbolic computation, Differentiation and Integration, Numerical solution of equations. Introduction to LaTeX, Syntax, Font Selection, Formatting Text, Page Layout and style, Table preparation, Equation writing, Bibliography with LaTeX.	40

Course Outcomes:

After studying these topics, the students will be able to

CO1: Understand the probability distributions.

CO2: Apply the probability distribution in inventory models.

CO3: Use Mathematica software to get the solution of real-life inventory models.

CO4: Write research article in LaTeX.

Text Books:

- S. C. Gupta and V. K. Kapoor, Fundamentals of Mathematical Statistics: Sultan Chand & Sons, 2014.
- S. Wolfram, The Mathematica book: Wolfram Media, Inc, 2004.
- S. Kottwitz, LaTeX beginner's Guide, Packt Publishing, 2011.

- ▶ G. Grätzer, More Math Into LaTeX, Springer, 2016.
- E. Don, Schaum's Outline of Mathematica and the Wolfram Language, Mc Graw Hill Education, 2018.



Mapping of Course Outcomes (COs) with Program Outcomes (POs) and Program Specific Outcomes (PSOs):

COs	POs/ PSOs
C01	PO1, PO10, , PO11 / PSO1, PSO4
C02	P010, P011 / PS01, PS04
C03	PO3, PO4 / PSO1, PSO2, PSO4
CO4	PO1, PO2, PO10, PO11 / PSO1, PSO3, PSO4



PMAE 0030: PARTIAL DIFFERENTIAL EQUATIONS: METHODS AND APPLICATIONS

Course Objectives: To make the students understand the various analytic and semi analytic approaches to solve linear and nonlinear partial differential equations with given initial and boundary conditions by giving more emphasis to their applications in the real world.

Cred	its:	04
		~ -

Semester I

L-T-P: 4-0-0

Module	Contents	Teaching Hours
No.		(Approx.)
	I Order Linear Partial Differential Equations (PDEs):	
	Solution by Adomian decomposition method (ADM), The	
	noise terms phenomenon. Solution by the modified	
	decomposition method (MDM) and the variational	
	iteration method (VIM). Method of characteristics,	
	Solution of systems of linear PDEs by ADM and VIM.	
	Heat Flow and Wave Equations by ADM and VIM:	
	Solution of homogeneous and inhomogeneous one	
	dimensional heat and wave equations by ADM and VIM.	45
Ι	Solution of higher dimensional heat flow & wave	45
	equations by ADM. Solution of Laplace's equation with	
	Dirichlet, Neumann and Robin Boundary conditions by	
	ADM.	
	Non - linear Partial Differential Equations: Calculation	
	of Adomian polynomials, Solution of nonlinear PDEs by	
	ADM, MDM and VIM, Solution of non-linear PDEs systems,	
	Nonlinear advection problem, Goursat problem, Klein-	
	Gordon, Sine-Gordon, Burger's, Telegraph, Schrodinger,	
	Korteweg – deVries (KdV) equations by ADM and VIM.	

Learning Outcomes:

After studying these topics, the student will be able to

- > Understand various numerical methods for solving differential equations
- > Apply the semi analytic methods in real world problems
- Explore new applications of the aforesaid methods
- > Solve the non-linear PDE numerically

Text Books:

- > A. M. Wazwaz, Partial Differential Equations and Solitary Wave Theory, Springer, 2009.
- A. M. Wazwaz, Partial Differential Equations: Methods and Applications, Balkema Publishers, 2002.

- G. Adomian, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Academic Publishers, 1994.
- L. Debnath, Non-Linear Partial Differential Equations for Scientists and Engineers, Birkhauser, Springer, 2012.
- > L. C. Evans, Partial Differential Equations, American Mathematical Society, 1998.



Course Curriculum (Session 2021-22) [Ph. D. (Mathematics) Course Work]

PMAE 0031: FRACTIONAL DIFFERENTIAL EQUATIONS

Course Objectives: To make the students understand the concepts of fractional integral, fractional derivative and solution of fractional differential equations.

Credits: 04

Semester I/II

L-T-P-J: 4-0-0-0

Module No.	Contents	Teaching Hours
		(Approx.)
No.	Special Functions – Euler's functions, Integral functions, One and two parameter Mittag-Leffler functions. Fractional Calculus – Introduction, definition, Fractional integral of order α , Grünwald–Letnikov fractional derivative, Riemann-Liouville (RL) fractional derivative of order α with its properties, Liouville- Caputo fractional derivative of order α with its properties, Laplace transform of fractional integrals and derivatives. Fractional Differential Equations (FDE) – Riemann-Liouville and Caputo fractional differential equations, Existence and uniqueness for the Caputo problem, linear and nonlinear fractional differential equation, Solution by Adomian decomposition method (ADM), Fractional systems of differential equations. Solution of Fractional Differential Equations – Numerical solution of FDE by fractional Hunter-Saxton equation, fractional Newell-Whitehead-Segel equation, fractional system of Bloch equations, fractional disease models etc., Hybrid methods for solving fractional differential equations, Application of homotopy perturbation transform method (HPTM) and homotopy analysis transform method (HATM) in solving time-fractional multi-dimensional telegraph equation etc. Use of any one software package for numerical	
	computation, and drawing figures.	

Course Outcomes:

After studying these topics, the students will be able to

- **CO1:** Understand the fractional integral and derivatives.
- **CO2**: Apply the numerical methods in solving fractional differential equations.
- **CO3:** Use hybrid methods to get the solution of real-life fractional nonlinear models.

CO4: Identify a research problem.

Text Books:

- C. Milici, G. Draganescu and J. T. Machado, Introduction to Fractional Differential Equations: Nonlinear Systems and Complexity, Springer Nature Switzerland AG, 2019.
- A. A. Kilbas, H. M. Srivastava, J. J. Trujillo, Theory and applications of fractional differential equations, Elsevier B.V., Amsterdam, 2006.

- > I. Podlubny, Fractional differential equations, Vol. 198, Academic Press, USA.
- E. Don, Schaum's Outline of Mathematica and the Wolfram Language, Mc Graw Hill Education, 2018.



Mapping of Course Outcomes (COs) with Program Outcomes (POs) and Program Specific Outcomes (PSOs):

COs	POs/ PSOs
C01	P01, P02, P03, P04 / PS01, PS04
CO2	PO2, PO3, PO4, PO10 / PSO2, PSO4
C03	PO2, PO3, PO4, PO10, PO12 / PSO2, PSO4
CO4	PO4, PO8, PO10 / PSO2, PSO3



Credits: 04

L-T-P: 4-0-0

PMAE 0040: COMPUTER BASED NUMERICAL TECHNIQUES AND SOFT COMPUTING

Course Objectives: To make the students understand the concepts of numerical techniques, artificial neural networks, genetic algorithm and fuzzy logic by giving more emphasis to their applications in soft computing.

Semester I

Module No.	Contents	Teaching Hours (Approx.)
Ι	 Introduction: Brief review & updates of concepts & applications of computer and computer programming, Modern trends. Numerical Techniques: Finding roots of equation & optimization, Least-square-fitting, Laplace-Transform and its applications for the numerical solution of differential equations, FEM and its applications. Neural Networks: Introduction to soft computing & Neural computing, Fundamentals of artificial neural network (ANN), Models of ANN, ANN models: Rosenblatt's perception, McCulloch & Pitts Model, Single layer perceptron, Learning methods in perceptron, Linearly separable task and XOR problem. Multi-Layer Perceptron: Back propagation learning algorithm, Associative memory, Hopfield network, Auto associative memory, Bidirectional hetro-associative memory, ADALINE, MADALINE network, Applications of neural network. Fuzzy Logic: Introduction to Fuzzy and Crisp sets, Fuzzy membership and Fuzzy operations, Properties of fuzzy sets, Linguistic hedges, Fuzzy logic – T-norms and other aggregation operators, Crisp and Fuzzy relations, Fuzzy system, Crisp logic, Propositional logic and its Laws, Inference in propositional logic, Fuzzy logic, Inference in Fuzzy logic (GMP and GMT), Fuzzy rule based system, Fuzzyfications & Defuzzifications, Applications of Fuzzy logic. Genetic Algorithm (GA): Introduction to GA, Search optimization method, Evolutionary algorithm working principles of GA, Encoding, operators of GA, Crossover and mutation, Basics of genetic algorithm with example for maximize, Introduction to genetic programming and Basic concepts. 	45

Learning Outcomes:

After studying these topics, the student will be able to

- > Implement code for numerical methods to find the solution of non-linear equations or the system of linear equations
- > Develop code for numerical methods to estimate the solution of higher order ordinary differential equations or systems of first order ordinary differential equations
- Construct graphical displays of science/engineering data and interpret the role of such displays in data analysis



- > Analyze various neural network architectures
- Define the fuzzy systems
- > Analyze the genetic algorithms and their applications

Text Books:

- S. Rajsekaran and G. A. Vijayalakshmi Pai, Neural Networks, Fuzzy Logic and Genetic Algorithm: Synthesis and Applications, Prentice Hall of India, 2011.
- Y. P. Kanetkar, Let us C, BPB publications, 2004.
- > V. Rajaraman, Computer oriented numerical methods, PHI Learning Pvt. Ltd., 2018.

- > T. J. Ross, Fuzzy Logic with Engineering Applications, John Wiley and Sons, 2016.
- D. E. Goldberg, Genetic Algorithm in Search Optimization and Machine Learning, Adission-Wesley, 2016.
- Karray, Soft Computing and Intelligent Systems Design: Theory, Tools and Applications, Pearson Education, 2009.



Head of the Department Mathematics Institute of Applied Sciences & Humanilies GLA University, Mathura

PMAE 0041: APPLIED STATISTICS AND OPTIMIZATION TECHNIQUES

Course Objectives: To make the students understand the concepts of statistics and optimization techniques by giving more emphasis to their applications in soft computing.

Credits: 04

Semester I

L-T-P: 4-0-0

Module	Contents	Teaching Hours
No.		(Approx.)
I	 Random Variables: Definition, Cumulative distribution function, Continuous, Discrete and Mixed random variables, Probability density function, Probability mass function. Moments of random variables, Mean and variance of random variable, Coefficients of variation, Skewness and Kurtosis, Moments, Covariance and Correlation coefficient. Properties of distribution functions. Probability Distributions and Testing of Hypothesis: Binomial, Poisson and Normal distributions. Introduction to Statistical analysis, Population distribution, Sampling and Non sampling Errors, Point estimation, Interval estimation, Student's t-distribution, Chi Square distribution problems, Classical optimization techniques: Single variable optimization, Multivariable optimization with no constraints, Multivariable optimization with equality constraints. Non-linear Programming: Minimization methods, Introduction, Exhaustive search, Direct root method, Newton method, Secant method, Unconstrained optimization techniques, Decision theory. Stochastic Processes: Definition and Classification of Stochastic processes, Poisson process, Birth and Death process, Applications to queues, Discrete time Markov chains. 	45

Learning Outcomes:

After studying these topics, the students will be able to

- Apply the concepts of probability, random variables and operation research in the area of computer networks, image processing etc.
- > Use statistical methods for the design and analysis of computer simulation experiments
- State and define the inference from small samples including difference between two means, paired differences, population variances, and two population variances
- Understand the characteristics of different types of decision-making environments and the appropriate decision-making approaches
- > Learn Poisson process and its applications in queuing theory



Text Books:

- K. S. Trivedi, Probability and Statistics with Reliability, Queuing and Computer Science Applications, Wiley, 2016.
- H. A. Taha, Operations Research An Introduction, Prentice Hall, 2011.

- Papoulis and S. U. Pillai, Probability, Random Variables and Stochastic Processes, Tata McGraw Hill, 2017.
- R. M. Gray and L. D. Davisson, An Introduction to Statistical Signal Processing, Cambridge University Press, 2004.
- S. L. Miller and D. G. Childers, Probability and Random Processes, Academic Press, 2012.
- Y. Viniotis, Probability and Random Processes for Electrical Engineers, McGraw Hill, 1998.
- S. D. Sharma, Computer Based Optimization Techniques , Kedr Nath Publication, 2011.
- K. Swaroop and Man Mohan, Operation Research, Sultan Chand & Sons, 2011.
- > D. S. Hira and P. K. Gupta, Operation Research, S. Chand Publication, 2005.

Course No:		cal Computation and Course Code: PMAE 0042								
Batch: NA	Programme: Ph. D. (Mathematics)	Semester:	L	Т	Р	J	Credits	Contact l Per Wee		
	r II. D. (Mathematics)	Ι	4	0	0	0	4	Total Ho	urs: 40	
Total Evalua	ation Marks: 100	Examinati	on D	urati	on: Ei	nd Te	rm (3 ho	urs)		
End Term: Internal As	30 Marks sessment : 20 Marks	Pre-requis	ite o	f cou	rse: N	IIL				
Course Objective	This course will develop a profound understanding of generating probability distributions in R, central limit theorem and random number generation. This course will also provide the knowledge of Gaussian integration, Monte Carlo integration, Bootstrapping and jackknife resampling. This course focuses on employability and skill development aligned with all CO's.									
Course Outcomes	After studying these topics, the students will be able to CO1 : Generate probability distributions in R and random number by various methods. CO2 : Apply central limit theorem. CO3 : Compute integrals using quadrature formula, Gaussian integration and Monte Carlo methods. CO4 : Know Bootstrapping for estimation of sampling distribution.									
	C	OURSE SYLI		S						
Module No.		Cont	ent						Hours	
I	Generating discrete and contir Distributions. Concept of Mark Random number generation Rejection method. Optimization: Newton-Raphs missing/incomplete data prob algorithm. Methods to compute integrals integration. Monte Carlo Simulation: Mo methods, Metropolis- Hasti Bootstrapping, jackknife resa distribution.	ov chains. Si n: General on algorithr lems and miz quadrature onte Carlo in ings and G	mula trans n, El cture form ntegra ibbs	ting m sforma M alg mode ula, do ation, samj	ultiva ation orithm ls, min buble i Mark pler	n winnoriza integr and	distribut hods, Ac th applic ation max cation, Ga chain Mo related	ions. ceptance- ations to imization ussian nte Carlo methods.	40	
 R. Chr Applic C. Dav Univer M. L. F Reference b W. J. K 	ella & C. P. Roberts, Monte Carl istensen, W. Johnson, A. Branso ations, Springer, 1996. ison, & D. V. Hinkley, Bootstrap sity Press, 1997. Sizzo, Statistical computing wit	cum & G. S. Fi o methods an h R, CRC Pres al computing	shma d the s, 20 Marc	in, Mo ir app 19. el Deł	nte Ca licatio	arlo: (on (No	Concepts, o. 1). Cam	U	ıs, and	

Course No:	NA Course Name: Advan	nced Deep learni	ng		Cour	se C	ode: PM		
Batch: NA					Contact H Per Wee				
		I	4	0	0	0	4	Total Ho	1rs: 40
Total Evalu	ation Marks: 100	Examinatio	n D	urati	on: Ei	nd T	erm (3 h	ours)	
End Term: 80 MarksPre-requisite of course: NInternal Assessment: 20 Marks									
Course Objective	This course provides an in-depth exploration of advanced neural network architectures, covering the theory and practical implementation of Convolutional Neural Networks (CNN), Recurrent Neural Networks (RNN), and Generative Adversarial Networks (GAN). Students will build and optimize models, with applications in various data science domains.								
Course Outcomes	 After studying these topics, CO1: Understand the concalgorithms behind th CO2: Develop proficiency in CO3: Grasp Convolutional implantation in image CO4: Understand RNNs, en and variants like LST CO5: Introduction to Generative 	cept of neural ne nem. n implementing a Neural Networ e processing. ncompassing the M and GRU	etwo and ks (han	ork the optim (CNNs) idling	eory, izing (), pop of sec	CNN, oular quent	RNN, and CNN arc	d GAN moc chitecture, RNN archi	lels. and i tectur
		COURSE SYLL	ABU	JS					1
Module No	•	Conte	nt						Hour
	Deep Neural Network: C challenges of deep networ operation, pooling layers (I (ReLU, Leaky ReLU), Regula batch normalization), fully VGG, ResNet etc., Concept of Image processing: Basics spaces, image rescaling, da	rks, Convolution Max, average, an rization Techniq connected layer f Transfer learnin	Ne dgl ues s, P ng a	eural lobal p (Drop opulat nd its	Netwo boolin bout, I bout, I bonef ta: pix	ork g), A L1 an I arc its, n kels,	(CNN), Ca activation ad L2 regu hitecture nodel eva	onvolution Functions Ilarization, s: AlexNet, lutaion.	
Ι	blurring, and sharpening, in Recurrent Neural Networ of RNN, Long short-term r gate, input gate, output gate backward propagation thro Generative Adversarial Ne GAN, Overview of generat	ata augmentatio nage data handlin ks (RNN): Introo nemory (LSTM), e, Gated recurren ugh time. e tworks (GAN): ive vs. discrimi	n, f ng b luct Are it ur Inti nati	ilters y CNN cion of chitect nit (GF roduct ve mo	E Sequ ture a RU), Fo tion to odels,	entia and o orwa o Ger	els: edge al data, An componen ard propa nerative M	detection, rchitecture nts: Forget gation and Iodels and	40
Text Books ➢ I, Go ➢ A. Ro	blurring, and sharpening, in Recurrent Neural Networ of RNN, Long short-term r gate, input gate, output gate backward propagation thro Generative Adversarial Ne GAN, Overview of generat GANs, Applications of generat : odfellow, Y. Bengio and A. Co osebrock, Deep Learning for G	ata augmentatio nage data handlin ks (RNN): Introd nemory (LSTM), e, Gated recurren ugh time. etworks (GAN): ive vs. discrimi ative models in o urville, Deep Lea Computer Vision	n, f ng b luct Arc t ur Intr nati <u>lata</u> rnin wit	ilters y CNN ion of chitect nit (GF roduct ve mo scient ng, Can h Pyth	E. Sequ ture a RU), Fo cion to odels, ce. mbrid on, Py	entia and c orwa o Ger Eva ge, M yIma	els: edge al data, An componen ard propa nerative M luation r 1A: MIT P geSearch	detection, rchitecture nts: Forget gation and Aodels and netrics for ress, 2016	40
Text Books ≻ I, Go ≻ A. Ro ≻ M. N Reference I	blurring, and sharpening, in Recurrent Neural Networ of RNN, Long short-term r gate, input gate, output gate backward propagation thro Generative Adversarial Ne GAN, Overview of generat GANs, Applications of gener : odfellow, Y. Bengio and A. Co psebrock, Deep Learning for G ielsen, Neural Networks and	ata augmentatio nage data handlin ks (RNN): Introd nemory (LSTM), e, Gated recurren ugh time. etworks (GAN): ive vs. discrimi ative models in c urville, Deep Lea Computer Vision Deep Learning. I	n, f ng b luct Aro t ur Intunati lata rnin wit	ilters y CNN cion of chitect nit (GF roduct ve mo <u>scieno</u> ng, Can h Pyth rmina	E Sequ ture a RU), Fe cion to odels, ce. mbrid on, Py tion F	entia and corwa o Ger Eva ge, N yIma Press	els: edge al data, An componen ard propa nerative M luation r 1A: MIT P geSearch , 2015.	detection, rchitecture nts: Forget gation and Aodels and netrics for ress, 2016 , 2017.	40



Head of the Department Mathematics Institute of Applied Sciences & Humanities GLA University, Mathura

PMAE 0050: DIFFERENTIAL GEOMETRY OF MANIFOLDS

Course Objectives: To make the students understand the concepts of surface, tangent vectors, tangent plane, curvature and differential manifold by giving more emphasis to their applications in differential geometry.

Credits: 04

Semester I

L-T-P: 4-0-0

Module	Contents	Teaching Hours
No.		(Approx.)
	Functions on Euclidean spaces, Continuity, Differentiability,	
	Partial and Directional derivatives, Chain rule, Inverse	
	function theorem, Implicit function theorem, Smooth	
	Urysohn lemma, Partition of unity, Change of variables.	
	Regular surfaces in R ³ , Coordinate neighbourhoods, Tangent	
	vectors, Tangent plane, Normal fields, Orientability,	
	Examples of surfaces, Level sets of smooth functions on R^3 .	
Ι	Smooth functions on surfaces, Differential of a smooth	45
	function, Gauss map, Shape operator (or Weingarten map),	
	Normal sections, Principal curvatures, Gaussian and Mean	
	curvature, Theorema Egregium, Isometries of surfaces.	
	Differential manifolds, Differential functions on manifolds,	
	Tangent spaces, Vector fields, Differential forms on	
	manifolds, Orientations, Integration on manifolds, Stoke's	
	theorem on manifolds	

Learning Outcomes:

After studying these topics, the student will be able to

- Understand change of variables
- > Know regular surfaces and orientability
- > Use Weingarten map, principal curvatures and isometries of surfaces
- > Solve the problems based on Gauss map, normal sections, Gaussian and mean curvature
- > Learn the concept of differential forms on manifolds and integration on manifolds

- > M. Spivak, Calculus on Manifolds, Addison-Wesley, 1965.
- J. R. Munkers, Analysis on Manifolds, Addison-Wesley, 1991.
- > A. Pressley, Elementary Differential Geometry, Springer, 2001.
- A. Gray, Modern Differential Geometry of Curves and Surfaces with Mathematica, CRC Press, 1997.



Head of the Department Mathematics Institute of Applied Sciences & Humanities GLA University, Mathura

PMAE 0051: RIEMANN-FINSLER GEOMETRY

Course Objectives: To make the students understand the concepts of Finsler metric, Chern connection, flag curvature, Riemann curvature, Parallel translation and Jacobi field by giving more emphasis to their applications in Finsler geometry.

Credits: 04

Semester II

L-T-P: 4-0-0

No.(Approx.)Minkowski norm, Finsler metric, Length Structure and Volume Form, Randers metric. Chern connection, Flag Curvature, Finsler metric of constant flag curvature, Finsler metric of scalar flag curvature.(Approx.)IProjective Finsler metric of constant flag curvature, Finsler metric of scalar flag curvature.45IParallel vector Fields, Parallel translation, Landsberg metric, Berwald metric.45IRiemann curvature, S-Curvature, Isotropic S-curvature, Exponential map, First and second variation, Jacobi field.45	Module	Contents	Teaching Hours
Volume Form, Randers metric. Chern connection, Flag Curvature, Finsler metric of constant flag curvature, Finsler metric of scalar flag curvature.Projective Finsler metric, Projective flat Finsler metrics, Parallel vector Fields, Parallel translation, Landsberg metric, Berwald metric.45IBerwald metric. Riemann curvature, S-Curvature, Isotropic S-curvature,45	No.		(Approx.)
Maple programs for Spray Coefficients of two dimensional Finsler metric, Gauss curvature, Spray Coefficients of (α, β)	No.	Minkowski norm, Finsler metric, Length Structure and Volume Form, Randers metric. Chern connection, Flag Curvature, Finsler metric of constant flag curvature, Finsler metric of scalar flag curvature. Projective Finsler metric, Projective flat Finsler metrics, Parallel vector Fields, Parallel translation, Landsberg metric, Berwald metric. Riemann curvature, S-Curvature, Isotropic S-curvature, Exponential map, First and second variation, Jacobi field. Maple programs for Spray Coefficients of two dimensional	(Approx.)

Learning Outcomes:

After studying these topics, the student will be able to

- > Know Finsler metric, Randers metric and Chern connection
- > Understand Finsler metric of constant and scalar flag curvatures.
- > Solve the problems based on projective and projective flat Finsler metrics
- Learn Landsberg and Berwald metrics
- Find parallel vector fields, parallel translation, Riemann curvature, S-curvature and Exponential map
- > Calculate spray coefficients and Gauss curvature using Maple

- Z. Shen, Differential Geometry of Spray and Finsler Space, Kluwer Academic publishers, 2001.
- Z. Shen, Lectures on Finsler Geometry, Lectures on Finsler Geometry, World Scientific, 2001.
- S. S. Chern and Z. Shen, Riemannian Finsler Geometry, World Scientific, 2004.
- D. Bao, S. S. Chern and Z. Shen, An Introduction to Riemannian-Finsler Geometry, GTM, Springer, 2000.
- P. L. Antonelli, R. S. Ingarden, M. Matsumoto, The Theory of Sprays and Finsler Spaces with Application in Physics and Biology, Kluwer Academic Publishers, 1993.
- M. Matsumoto, Foundations of Finsler Geometry and Special Finsler spaces, Kaiseisha Press, Japan 1986.

Course No:	: NA Course Name: Algebraic Graph Theory Course Code: PMAE 0052													
Batch: NA		Programme:	Semester: I	L	Т	Γ P J Credits Contact Hrs Per Week: 4								
		Ph. D. (Mathematics)		4	0									
Fotal Evalu	ation	Marks: 100	Examinatio	n D	ura	tio	n: E	nd Te	rm	(3 hours)				
End Term: 80 Marks Pre-requisite of course:														
	sessr	nent: 20 Marks		vith	Abs	trac	t Al	gebra	and	l Linear Algebra				
Course Objective	an alg algeb stude	his course will develop an in-depth understanding of the principles of graph theory from n algebraic perspective. The students will be able to analyze graph properties using gebraic methods such as group theory and matrix theory. This course will also help the rudents to explore advanced topics in algebraic graph theory, including graph utomorphisms and eigenvalues.												
Course Outcomes	CO1: CO2: CO3: CO4:	studying these topics, t Utilize algebraic structu Analyze the spectral pr Compute chromatic algebraic methods. Develop the ability to a areas, such as network Conduct independent algebraic graph theory,	ires to represe operties of gra polynomials a pply algebraic theory and sp research and	ent a phs ind gra ectr pro	and ana ph t al cl eser	anal alyz theo lust it fi	yze e gr ery te ering ndin	aph o echniq g. lgs on	colo jues ac	s in advanced re lvanced topics	search			
			COURSE SYLI	AB	US									
Module No.			Cont	ent							Hour			
I	trees Alge Lapl Eige math Spee Chee App colo grap Alge	ph Theory: Introductions, and graph representate braic Structures in acian matrix and its pro- roces, Spectral propertiens ctral Graph Theory: eger's inequality, Rando lications of Algebraic ring, Graph isomorphic braic connectivity and lications in network the	tions. Graphs: Adj perties, The M Spectra: Eige s of graphs, Co The Laplacia m walks on gra Graph Theo sms and graj d Fiedler veo	acer atri enva spe n s aph ry: oh i	ncy x-Ti alue ctra spec s an Chr reco s, S	and ree s o l gra trun d th omst onst	d in theo f ac aphs n a eir s atic ructi tral	iciden rem a ljaceno and t nd gr pectra polyn ion pi clust	ce nd a cy hei aph al ir om rob com	matrices, The applications. and Laplacian r significance. n connectivity, nterpretation. ials and graph lem, Expander	20			
Text Books ≻ N. Big		lgebraic Graph Theory, (-			-								
🕨 R. Die	R. K. C estel,	: hung, Spectral Graph Th Graph Theory, Springer, 5, Modern Graph Theory	2017.		athe	ema	tical	Socie	ty,	1996.				

Course No:	NA	Course Name: Advar	nced Algebraic Graph Th				ours			
Batch: NA		Programme:	Semester: I	L	Т	ł	P J	Credits	Contact	
		Ph. D. (Mathematics)		_	0			-	Per We	
				4	0		0 0		Total Ho	ours: 40
Total Evalua	ation	Marks: 100	Examination Duration		End	l l	ſerm	(3 hours)	
End Term:	80 Ma	arks	Pre-requisite of cour s A solid foundation in Lin			1	ahna	Abatraat	Algohno	md
Internal As	sessn	nent: 20 Marks	Basic Graph Theory.	iea		ıge	ebia,	ADSUACE	Algebia, a	mu
Course	comp Angs	onents graphs, and suman Das. The studen	a comprehensive under ubspace sum graphs, ind its will be able to learn t	cluo he i	ding fou	g no	the t datio	heoretical nal princi	contribu ples behin	tions of nd these
Objective	Evnes of graphs and their applications in vector spaces and algebraic structures. This co								graphs,	
Course Outcomes	CO1: CO2: CO3:	Construct and analyz sum graphs based on Understand and app number) of these gray Explore application combinatorics.	the students will be able e subspace inclusion gra the work of Angsuman I oly the structural prope phs. is of these graphs in isor graphs from com	aph Das erti 1 V	s, c es ecto	(c	degre r spa	ee, connec aces, line	ctivity, ch ar algeb	romatic ra, and
			COURSE SYLLABUS							
Module No.			Content							Hours
I Text Books:	subsp Revie Prop graph Struc Subs Prop Zero comm Struc Adva isome	baces. Construction of ew of Angsuman Das' of erties of Subspace In a coloring in subspace ponents Graphs: Defitural properties: conn pace Sum Graphs: erties of subspace sum Divisor Graphs: Intro- nutative rings, Constri- tural properties: graph nced Topics in Alg	hs: Introduction, Basic subspace inclusion grap contributions to subspace inclusion Graphs: Degr inclusion graphs. inition and construction ectedness, cycles, and in Subspace sums and the graphs: chromatic num oduction, Basic concepts uction of zero divisor g h connectivity, diameter gebraic Graph Theory inclusion, union, and su	ohs e in ree n o dep neir ber in r grap , an ; G	bass clu seq f cc pen r re r re r re bhs d c rap	se isi qui on ide ep raj g t fr ;yc	d on on gr ences npon ent se ph iso heor rom cles.	inclusion raphs. s, connect ents unio ets. ntation a omorphise y: zero div commutat	relations ivity, and n graphs s graphs m. visors and ive rings	40
C. GoN. Big	dsil aı ggs, Al		Graph Theory, Springer, 7, Cambridge University l er, 2017.)12.			
Alget > A. Da	Andei ora, Vo is, No	olume 217, Issue 2, Pa	aph of a Finite Dimensi	-						



Course Curriculum (Session 2020-21) [Ph. D. (Mathematics) Course Work)]

PMAE 0060: FLUID DYNAMICS AND SOFTWARE TOOLS

Course Objectives: To make the students understand the concepts of fluid, its properties and behavior under various conditions of internal and external flows.

Credits: 04	4 Semester I L-T-P-J	I: 4-0-0-
Module No.	Contents	Teaching Hours (Approx.)
Re am pr Re Gu mu sc de (i) sta (i) sta (i) sta di nu sta re Tu tu tu tu	asic Concepts of Fluid Dynamics: eal fluids and ideal fluids, stream lines, steak lines and path lines, steady nd unsteady flows, Lagrangian and Eulerian description, fundamental rinciples of conservation, conservation of mass, equation of continuity, eynolds transport theorem. overning Equations of Fluid Dynamics: The continuity equation, omentum equation, energy equation, conservation of linear omentum, Navier-Stokes equation, conservation of energy, general ralar transport equation, Boussinesq approximation, some solvable fully eveloped viscous flow:) steady flow between parallel plates, (ii) steady flow in a pipe, (iii) eady flows between concentric cylinders. imensional Analysis eynolds law of similarity, physical signification of some non- mensional parameters: Prandtl number, Mach number, Reynolds umber, Rayleigh number, Grashof number, Nusselt number, Laminar eady flow of incompressible viscous fluids in tubes of circular and ectangle cross- section. urbulence and its Modelling: Characteristics of turbulence, effect of urbulent fluctuations on mean flow, turbulent flow calculations, urbulence modelling. oftware Tools: troduction to software: MATHEMATICA and MAPLE. Introduction to aTeX, Syntax, Font Selection, Formatting Text, Page Layout and style, able preparation, Equation writing, Bibliography with LaTeX.	40

Course Outcomes:

After studying these topics, the students will be able to

- **CO1:** Understand physical properties of a fluid.
- **CO2**: Know the classification of the basic equations of fluid dynamics.
- **CO3:** Recognize the type of fluid flow occurring in a particular physical system and to use the appropriate model equations to investigate the flow.
- **CO4:** Learn the dimensional analysis.
- **CO5:** Write the research article in LaTeX.

Text Books:

- > P. G. Drazin and W. H. Reid, Hydrodynamic Stability, Cambridge University Press, 2004.
- S.V. Parankar, Numerical Heat Transfer and Fluid Flow, McGraw-Hill. 1980.
- K. Muralidhar and G. Biswas, Advanced Engineering Fluid Mechanics, Narosa Publishing House, 2006.
- G. D. Smith, Numerical Solution of Partial Differential Equations: Finite Difference Methods, Clarendon Press, 1985.
- S. Wolfram, The Mathematica book: Wolfram Media, Inc, 2004.
- S. Kottwitz, LaTeX beginner's Guide, Packt Publishing, 2011.

Reference Books:

> H. Schlichting, Boundary Layer Theory, McGraw Hill Education, 2014.



Mapping of Course Outcomes (COs) with Program Outcomes (POs) and Program Specific Outcomes (PSOs):

COs	POs/ PSOs
C01	P01, P02, P04, P06, P010, / PS01, PS02, PS04
C02	P01, P02, P03, P04, P06, P09, P012 / PS01, PS02, PS04
C03	P01, P02, P03, P04, P05, P06, P010, P012 / PS01, PS02, PS03, PS04
CO4	P01, P02, P03, P04, P010, P011, P012/ PS01, PS02, PS04
C05	P01, P02, P010, P011 / PS01, PS03, PS04



Course Curriculum (Session 2020-21) [Ph. D. (Mathematics) Course Work)]

PMAE 0061: COMPUTATIONAL METHODS FOR FLUID DYNAMICS

Course Objectives: To make the students understand the concepts of stepwise procedure to completely solve a fluid dynamics problem using computational methods.

Credits: 04

Semester I/II

L-T-P-J: 4-0-0-0

Module	Contents	Teaching
No.		Hours
		(Approx.)
Ι	Introduction to Computational Fluid Dynamics (CFD): Advantages and applications of computational field, governing equations for CFD, continuity equation, momentum equation, energy equation, additional equations for turbulent flow. CFD Techniques: Discretization of governing equations, Finite-difference method, finite-volume method, converting governing equation to algebraic equation system, numerical solutions to algebraic equations-Direct method, iterative methods, pressure velocity coupling. Hydrodynamic Stability: Mathematical formulation of the stability problem of incompressible flow, method of normal modes, Squire's theorem and Orr-Sommerfeld equation; concept of boundary layers, fluid flow through a porous medium; porosity, permeability, Darcy law and its extensions.	40

Course Outcomes:

After studying these topics, the students will be able to

- **CO1:** Understand CFD technique.
- **CO2**: Demonstrate the ability to analyze a flow field to determine various quantities of interest, such as flow rates, heat fluxes, pressure drops, losses, etc., using flow visualization and analysis tools.
- CO3: Learn the hydrodynamics stability
- **CO4:** Simulate simple CFD models and analyze its results.

Text Books:

- J. Tu, G. Heng Yeoh and C. Liu Computational Fluid Dynamics: A practical Approach, Butterworth-Heinemann, 2018.
- S.V. Parankar, Numerical Heat Transfer and Fluid Flow, McGraw-Hill. 1980.
- > T. J. Chung, Computational Fluid Dynamics, Cambridge University Press, 2010.
- J. H. Ferziger and M. Peric, Computational Methods for Fluid Dynamics, Springer, 2002.
- G. D. Smith, Numerical Solution of Partial Differential Equations: Finite Difference Methods, Clarendon Press, 1985.

- H. K. Versteeg and W. Malalasekera, An Introduction to Computational Fluid Dynamics: The Finite Volume Method, Pearson, 2010.
- J. C. Tannehill, D. A. Anderson and R. H. Pletcher, Computational Fluid Mechanics and Heat Transfer, McGraw Hill, 2002.



Mapping of Course Outcomes (COs) with Program Outcomes (POs) and Program Specific Outcomes (PSOs):

COs	POs/ PSOs
C01	PO1, PO2, PO4, PO5, PO6, PO10, PO11/ PSO1, PSO2, PSO4
CO2	PO1, PO2, PO3, PO4, PO6, PO9, PO12 / PSO1, PSO2, PSO4
CO3	PO1, PO2, PO3, PO4, PO5, PO6, PO12 / PSO1, PSO2, PSO4
C04	PO1, PO2, PO3, PO4, PO10, PO11, PO12/ PSO1, PSO2, PSO4



Course Curriculum (Session 2020-21) [Ph. D. (Mathematics) Course Work)]

PMAE 0070: FOUNDATIONS OF COMPLEX ANALYSIS-I

Course Objectives: To make the students understand the foundations for functions of a complex variable and their applications.

Credits: 04 Semester I L-T-P-J: 4-0-0-0

Module No.	Contents	Teaching Hours (Approx.)
Ι	 Functions of a complex variable, Spherical representation of complex numbers, Limits, Theorems on limit, Continuous functions, Differentiability, Cauchy-Riemann equations, Polar form of Cauchy Riemann equations, Analytic functions, Harmonic functions. Definite integral, Line integral, Cauchy's theorem, Morera's theorem, Cauchy's integral formula, Higher derivatives, Liouville's theorem, Fundamental theorem of algebra, Maximum modulus theorem. Sequences and Series, Sequences and Series of functions, Power series, Taylor's theorem, Singularities, Laurent's theorem, Residues, Exponential and trigonometric functions, Contour integrations. Computer Skills: Introduction to LaTeX and MAPLE, Power point presentation through beamer. 	40

Course Outcomes:

After studying these topics, the students will be able to

- **CO1:** Use the information needed to prove theorems and establish mathematical results.
- **CO2**: Demonstrate the ability to integrate knowledge and ideas of complex differentiation and complex integration in a coherent and meaningful manner and use appropriate techniques.
- **CO3:** Demonstrate the ability to think critically by proving mathematical conjectures and establishing theorems from complex analysis.
- **CO4:** Use Latex in article writing and power point presentation.

Text Books:

- V. Ruel Churchill and J. W. Brown, Complex Variables and Applications, McGraw-Hill Publishing Company, 2013.
- S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 2011.
- > H. A. Priestly, Introduction to Complex Analysis, Clarendon Press, 2006
- J. B. Conway, Functions of one Complex Variable, Springer-Verlag, Narosa Publishing House, 1995.
- L. V. Ahlfors, Complex Analysis, McGraw Hill Education, 2017.

- S. Lang, Complex Analysis, Addison Wesley, 1977.
- M. J. Ablowitz and A. S. Fokas, Complex Variables: Introduction and Applications, Cambridge University Press, 2003.
- > W. Rudin, Real and Complex Analysis, Mc Graw Hill Education, 2017.



Mapping of Course Outcomes (COs) with Program Outcomes (POs) and Program Specific Outcomes (PSOs):

COs	POs/ PSOs
C01	PO1, PO2, PO4, PO5, PO6, PO10, PO11/ PSO1, PSO2, PSO4
C02	PO1, PO2, PO3, PO4, PO6, PO9, PO12 / PSO1, PSO2, PSO4
CO3	PO1, PO2, PO3, PO4, PO5, PO6, PO12 / PSO1, PSO2, PSO4
CO4	PO1, PO2, PO3, PO4, PO10, PO11, PO12/ PSO1, PSO2, PSO4



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Course Curriculum (Session 2021-22) [Ph. D. (Mathematics) Course Work)]

PMAE 0071: FOUNDATIONS OF COMPLEX ANALYSIS-II

Course Objectives: To make the students understand the concepts of contemporary complex analysis and to use the methods of complex analysis in other areas of mathematics.

Credits: 04

Semester I/II

L-T-P-J: 4-0-0-0

Module No.	Contents	Teaching Hours (Approx.)
Ι	Linear Functions, Function 1/z, Bilinear transformations, their properties and classification, Functions z ² and z ^{1/2} , Transformations w=exp(z) and w= sin z, Conformal mappings, Harmonic conjugate, Transformation of Harmonic Functions, Transformation of Boundary Conditions, Steady temperatures, Steady temperature in a half plane and related problems. Schwarz's lemma, Meromorphic functions, Argument principle, Rouche's theorem, Inverse function theorem. Open mapping theorem and Hurwitz' theorem, Riemann mapping theorem (Statement only), Area Theorem, Biberbach Theorem and conjecture, Koebe ¹ / ₄ theorem. Univalent functions, Star-like, Convex and Close-to-Convex functions.	40

Course Outcomes:

After studying these topics, the students will be able to

- **CO1:** Use the information needed to prove theorems and establish mathematical results.
- **CO2**: Learn the basic techniques of contemporary complex analysis as well as use of these techniques in various applications such as harmonic analysis, univalent functions theory and special functions.
- **CO3:** Demonstrate the ability to think critically by proving mathematical conjectures and establishing theorems from complex analysis.
- **CO4:** Demonstrate the ability to integrate knowledge and ideas of Meromorphic functions Univalent functions and Riemann mapping theorem in a coherent and meaningful manner and use appropriate techniques.

Text Books:

- V. Ruel Churchill and J. W. Brown, Complex Variables and Applications, McGraw-Hill Publishing Company, 2013.
- S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 2011.
- > H. A. Priestly, Introduction to Complex Analysis, Clarendon Press, 2006
- P. L. Duren, Univalent Functions, Springer-Verlag, New York, 1983.
- A. W. Goodman, Univalent Functions, Mariner Publishing Company, Michigan, 1983
- L. V. Ahlfors, Complex Analysis, McGraw Hill Education, 2017.

- S. Lang, Complex Analysis, Addison Wesley, 1977.
- M. J. Ablowitz and A. S. Fokas, Complex Variables: Introduction and Applications, Cambridge University Press, 2003.
- > W. Rudin, Real and Complex Analysis, Mc Graw Hill Education, 2017.



Mapping of Course Outcomes (COs) with Program Outcomes (POs) and Program Specific Outcomes (PSOs):

COs	POs/ PSOs
C01	PO1, PO2, PO4, PO6, PO10, PO11/ PSO1, PSO2, PSO4
CO2	P01, P02, P03, P04, P06, P09, P010, P011, P012 / PS01, PS02, PS04
CO3	P01, P02, P03, P04, P05, P06, P010, P011, P012 / PS01, PS02, PS03, PS04
CO4	P01, P02, P03, P04, P010, P011, P012/ PS01, PS02, PS04